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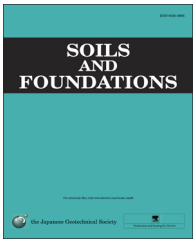

Prediction of recompression index using GMDH-type neural network based on geotechnical soil properties

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Abstract

Settlement based design for shallow foundations realizing the consolidation aspect is a major challenge in geotechnical engineering. The recompression index (C_r) from the oedometer test is used to estimate the consolidation settlement of over-consolidated (OC) clays. Since the determination of C_r from oedometer tests is relatively time-consuming and is usually determined for a single unloading, empirical equations based on index properties can be useful for settlement estimation. Correlations have been proposed to relate the C_r of clay deposits to other soil parameters. Since existing equations are incapable of estimating C_r well, artificial intelligence methods are used to predict them. In the present study, a Group Method of Data Handling (GMDH) type neural network is used to estimate the C_r from more simply determined index properties such as the liquid limit (LL) and initial void ratio (e_0) as well as specific gravity (G_s). In order to assess the merits of the proposed approach, a database containing 344 data sets has been compiled from case histories via geotechnical investigation sites in the province of Mazandaran, along the southern shoreline of the Caspian Sea, Iran. In addition to the physical properties mentioned already, the natural water content (ω_n), plastic index (PI) and dry density (γ_d) were also included in the model development. A comparison was carried out between the experimentally measured recompression indexes and the predictions in order to evaluate the performance of the GMDH neural network method. The results demonstrate that an improvement with respect to the other correlations has been achieved. Finally, a sensitivity analysis of the obtained model was performed to study the influence of the input parameters on the model output. The sensitivity analysis reveals that e_0 and LL have significant influence on predicting C_r.

Keywords: Recompression index; Consolidation settlement; Saturated clays; GMDH

1. Introduction

Geo-materials are extremely complex in terms of their stress–strain–time dependent behavior. These are due to soil non-linear stress–strain relationships, the time dependent response to loading, the elasto-plastic performance under the loading and unloading situation and the effects of stress history (pre-consolidation). For any earthen structure, a transition element is used to carry the loads from the super structure to the substructure or naturally deposited materials. The bearing capacity, settlement and structural design are the backbone of foundation engineering practice. Among three common occurrence settlement components, i.e. immediate, creep and consolidation time dependents, the latter plays an important role in

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Nomenclature

$\boldsymbol{a}$	Vector of unknown coefficients of the quadratic polynomial equation
a_i	Coefficients of the quadratic polynomial equation
C_{mi}	Actual measured output
C_{pi}	Predicted output
C_c	Compression index
C_r	Recompression index
E	Mean square error
e_0	Initial void ratio
G_i	Quadratic function
G_s	Specific gravity of soil particles
H	Initial thickness of the soil layer
i	Number of the value considered in cumulative probability P
LL	Liquid limit (%)
M	Total numbers of data sets
MAD	Mean absolute deviation
MAPE	Mean absolute percent error
n	Total numbers of input variables
PI	Plastic index (%)
P	Cumulative probability
RMSE	Root mean square error
R^2	Absolute fraction of variance
S_c	Primary consolidation settlement
X	Input variable
x_i	Input vector
Y	Vector of output's value from observation
y	Actual output
$\hat{y}$	Predicted output
σ_c'	Over consolidation pressure
σ_{v0}'	Initial vertical effective stress
$\Delta\sigma_v$	Load increment
ω_n	Natural water content (%)
γ_d	Dry density

geotechnical engineering (Holtz et al., 1986, 2011; Burland, 1990; Budhu, 2007).

The ability to predict settlement in saturated clays, especially the time dependent type of settlement known as consolidation, is an important issue in geotechnical engineering. Several techniques have been developed to predict settlement, including probabilistic measurements, analytical methods, regression analysis and simplified methods (Skempton and Sowa, 1963).

To calculate settlement for clays, laboratory consolidation tests which depict one-dimensional compression behavior need to be performed on samples taken from representative values (Terzaghi et al., 1996).

In settlement calculation for clays, in the case of normally consolidated (NC) condition, only the compression index (C_c) from the conventional oedometer test is required. If the clay is over consolidated (OC), then both compression and recompression indices (C_r) are necessary. C_r must be obtained to compute the level of settlement for OC clays as opposed to NC clays (Fig. 1).

For an NC clay deposit, the settlement due to an increase in load can be determined from the following equation:

$$S_c = \frac{C_c H}{1+e_0}\log\left(\frac{\sigma_{v0}'+\Delta\sigma_v}{\sigma_{v0}'}\right) \quad (1)$$

In OC clays, if $\sigma_{v0}'+\Delta\sigma_v \le \sigma_c'$:

$$S_c = \frac{C_r H}{1+e_0}\log\left(\frac{\sigma_{v0}'+\Delta\sigma_v}{\sigma_{v0}'}\right) \quad (2)$$

And $\sigma_{v0}'+\Delta\sigma_v > \sigma_c'$ then:

$$S_c = \frac{C_r H}{1+e_0}\log\left(\frac{\sigma_c'}{\sigma_{v0}'}\right)+\frac{C_c H}{1+e_0}\log\left(\frac{\sigma_{v0}'+\Delta\sigma_v}{\sigma_c'}\right) \quad (3)$$

where e_0 = initial void ratio, $\Delta\sigma_v$ = load increment, σ_c' = preconsolidation pressure, σ_{v0}' = initial vertical effective stress, C_c = compression index and C_r = recompression index.

In spite of the fact that the order of magnitude of the settlement of NC soils is approximately ten times greater than OC soil, and as a result the coefficient C_c may appear to be of more significance than C_r, this study concentrates on the development of correlations between C_r and the physical properties of soils, since the upper layers of soil adjacent to foundations are often over-consolidated. Furthermore, with the increased usage of as advanced constitutive models in numerical packages, the development of such correlations to arrive at better estimates of the coefficient of recompression is required. It is noteworthy that C_r is both stress and density dependent. Thus, substituting this concept with the linear elastic model is a great over-simplification.

As the oedometer test in the laboratory takes much longer to complete than simpler index property tests, various attempts have been made to estimate the C_r from other geotechnical tests which can be carried out more easily. Many researchers have used single parameter models to estimate C_r, i.e., liquid limit (LL), plastic index (PI), natural water content (ω_n), initial void ratio (e_0) and dry density (γ_d). However, others recommend multiple soil

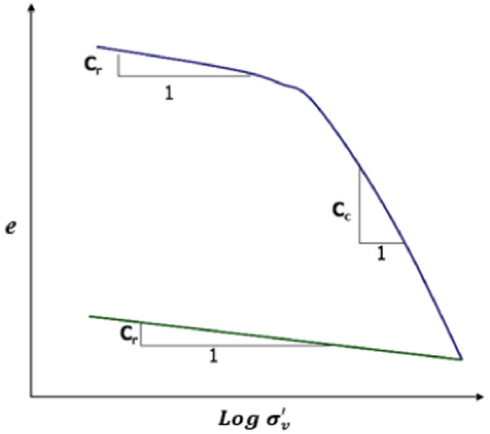

Fig. 1. Definition of C_c and C_r indices.

parameter models for the estimation of C_r. As presented in Table 1, several types of empirical correlations (one and multi-variable equations) have been selected. Moreover, easily obtainable fundamental characteristics of soils, which are of the same origin and/or from the same area, can be used to find the C_r indices of fine grained soils by these formulas.

To the authors' knowledge, there is no established theoretical relation between C_r and physical soil properties (e.g. liquid limit); thus, any attempt to establish an empirical correlation must be based on a general statistical analysis and system identification techniques. The interdependency of factors involved in such problems prevents the use of a simple regression analysis and demands a more extensive and elaborate method.

The ANN technique is well suited to the modeling of complex problems with an unknown relationship between the model variables. The advantage of the ANN is that it is very useful in learning the complex relationships between multi-dimensional data. Recently ANNs have been employed to model complex relationships between input and output data sets in geotechnical engineering (Uncuoglu et al., 2008; Ornek et al., 2012; Kalantary and Kordnaeij, 2012; Tarawneh, 2013; Shahin, 2014; Ochmanski et al., 2015). However, the main disadvantage of traditional ANN is that the detected dependencies are hidden within the ANN structure (Nariman-Zadeh et al., 2003).

The Group Method of Data Handling (GMDH) type neural network (NN) is a powerful identification technique and can be used to model complex systems, where unknown relationships exist between variables, without having specific knowledge of processes.

The present study aims to develop a GMDH-type NN for the prediction of C_r based on various soil parameters, such as e_0, LL and G_s without carrying out consolidation tests. Finally, the result of proposed method is compared with that of empirical equations referenced to measure the compression index.

2. Background to current methods

The aim of the Group Method of Data Handling (GMDH) is to identify the functional structure of a model hidden in empirical data (Ivakhnenko, 1971). The GMDH algorithm is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input-single-output data pairs. Over the last few years, GMDH-type neural networks have been applied to many geotechnical engineering problems with some degree of success. In this field, Kalantary et al. (2009), Ardalan et al. (2009), Shooshpasha and Mola-Abasi (2012), Mola-Abasi et al. (2013) and Eslami et al. (2014) applied GMDH model in predicting the undrained shear strength of clays, pile bearing capacity, liquefaction induced lateral displacement, shear wave velocity and the liquefaction potential based on the geotechnical soil properties, respectively. Therefore, this approach can be used in the empirical correlation of C_r as a step forward in comparison with statistical approaches.

3. Review of GMDH-type neural networks model

The concept of an artificial neural network (ANN) was inspired by the complex architecture of the human brain, regarded as a highly non-linear, parallel operating system (Haykin, 1999). Group Method of Data Handling (GMDH) type neural network is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input single-output data pairs $(X_i\ ,\ y_i)$ $(=1,\ 2,\ldots,\ M)$. The main idea of GMDH is to build an analytical function in a feed forward network based on a quadratic node transfer function whose coefficients are obtained using the regression technique. Using the GMDH algorithm, a model can be represented as a set of neurons in which different pairs in each layer are connected through a quadratic polynomial, producing new neurons in the next layer. Such representation can be used to map inputs to outputs. The formal definition of the identification problem is to find a function $\hat{f}$ that can be approximately used instead of the actual one, f in order to predict output $\hat{y}$ for a given input vector $X=(x_1,\ x_2,\ x_3,\ \ldots,x_n)$ as close as possible to the actual output y. Therefore, given M observation

Table 1
Some widely used recompression index equations.

Equation	Eq. no.	Reference
$C_r=0.0003(\omega_n+7)$	(4)	Azzouz et al. (1976)
$C_r=0.015(e_0+0.007)$	(5)	Azzouz et al. (1976)
$C_r=0.041-0.0268\ e_0$	(6)	Gunduz and Arman (2007)
$C_r=0.045-0.0283\ e_0$	(7)	Gunduz and Arman (2007)
$C_r=0.126-0.115\ e_0$	(8)	Gunduz and Arman (2007)
$C_r=0.00194(PI-4.6)$	(9)	Nakase et al. (1988)
$C_r=0.0007\ LL+0.0062$	(10)	Sinan (2009)
$C_r=0.1257\ \gamma_d^{-2.8826}$	(11)	Sinan (2009)
$C_r=0.126(e_0+0.003\ LL-0.06)$	(12)	Azzouz et al. (1976)
$C_r=0.0133\ e^{0.036\ \omega_n}$	(13)	Sinan (2009)
$C_r=0.000463\ LL\ G_s$	(14)	Nagaraj and Murty (1985)
$C_r=-0.000319\ \omega_n-0.027277\ \gamma_d+0.064019\ e_0+0.037$	(15)	Sinan (2009)

of multi-input-single-output data pairs (Farlow, 1984):

$$y_i = f(x_{i1},\ x_{i2},\ x_{i3},\ \ldots, x_{in}) \quad , \quad (i = 1\ ,2,\ 3,\ \ldots,\ M) \tag{16}$$

It is now possible to train a GMDH type of artificial neural network to predict the output values $\hat{y}_i$ for any given input vector $X = (x_{i1},\ x_{i2},\ x_{i3},\ \ldots, x_{in})$, that is:

$$\hat{y}_i = \hat{f}(x_{i1},\ x_{i2},\ x_{i3},\ \ldots, x_{in}) \quad (i = 1\ ,2,\ 3,\ \ldots,\ M) \tag{17}$$

The problem is now to determine a GMDH type of artificial neural network so that the square of the differences between the observed and predicted output is minimized as follow:

$$\sum_{i=1}^{M} \left[\hat{f}(x_{i1},\ x_{i2},\ x_{i3},\ \ldots, x_{in}) - y_i\right]^2 \rightarrow \min \tag{18}$$

The general connection between input and output variables can be expressed by a complicated discrete form of the Volterra functional series, known as the Kolmogorov-Gabor polynomial. Hence:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \ldots \tag{19}$$

This full form of mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of:

$$\hat{y} = G(x_i,\ x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \tag{20}$$

By this means, the partial quadratic description is recursively used in a network of connected neurons to build the general mathematical relation between inputs and output given in Eq. (19). The coefficients a_i in Eq. (20) are calculated using regression techniques to minimize the difference between the observed output, y and the calculated one, $\hat{y}$ for each pair of x_i, x_j as input variables (Jamali et al., 2009). Apparently, a tree of polynomials is constructed using the quadratic form given in Eq. (20) whose coefficients are obtained in a least squares scheme. In this way, the coefficients of each quadratic function G_i are obtained for an optimal fit for the output in the whole set of input–output data pairs, that is:

$$E = \frac{\sum_{i=1}^{M} \left(y_i - G_i(x_i, x_j)\right)^2}{M} \rightarrow min \tag{21}$$

In the basic form of the GMDH algorithm, all the possibilities of two independent variables out of the total n input variables are taken in order to construct the regression polynomial in the form of Eq. (20) that best fits the dependent observations $\left(y_i\ ,\ i = 1,\ 2,\ \ldots,\ M\right)$ in a least squares sense. Consequently $\binom{n}{2} = \frac{n(n-2)}{2}$ neurons will be built up in the first hidden layer of the feed forward network from the observations $\left\{\left(y_i,\ x_{ip}, x_{iq}\right); (i = 1,\ 2,\ 3, \ldots,\ M)\right\}$ for different $p,\ q \in \{1,\ 2,\ 3,\ \ldots,\ n\}$. In other words, it is now possible to construct M data triples $\left\{\left(y_i,\ x_{ip}, x_{iq}\right); (i = 1,\ 2,\ 3, \ldots,\ M)\right\}$ from observations using $p,\ q \in \{1,\ 2,\ 3,\ \ldots,\ n\}$ in the form of (Jamali et al., 2009)

$$\begin{bmatrix} x_{1p} & x_{1q} & \vdots & y_1 \\ x_{2p} & x_{2q} & \vdots & y_2 \\ \ldots & \ldots & \vdots & \ldots \\ x_{Mp} & x_{Mq} & \vdots & y_M \end{bmatrix}$$

Using the quadratic sub-expression in the form of Eq. (20) for each row of M data triples, the following matrix equation can be readily obtained as:

$$A\boldsymbol{a} = Y \tag{22}$$

$$\boldsymbol{a} = \{a_0, a_1, a_2, a_3, a_4, a_5\} \tag{23}$$

$$Y = \left\{y_1, y_2, y_3, \ldots, y_M\right\}^T \tag{24}$$

It can be readily seen that:

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1q}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & x_{Mp} & x_{Mp} & x_{Mq}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix} \tag{25}$$

The least squares technique from the multiple regression analysis leads to solution of the normal equations, in the form of:

$$\boldsymbol{a} = \left(A^T A\right)^{-1} A^T Y \tag{26}$$

This determines the vector of the best coefficients of the quadratic Eq. (20) for the whole set of M data triples. Notice that this procedure is repeated for each neuron of the next hidden layer according to the connectivity topology of the network. However, such a solution directly from normal equations is rather susceptible to the rounding off of errors and, more importantly, to the singularity of these equations.

There are two main concepts involved within GMDH type of artificial neural networks design, namely, the parametric and the structural identification problems. Nariman-Zadeh et al. (2003) presents hybrid genetic algorithm (GA) and singular value decomposition (SVD) method to optimally design such polynomial neural networks.

3.1. Application of GA in the topology design of GMDH-type neural networks

Stochastic methods are commonly used in the training of neural networks in terms of associated weights or coefficients and have successfully performed better than traditional gradient-based techniques (Atashkari et al., 2007). The literature shows a wide range of evolutionary design approaches either for architecture or connection weights separately, in addition to efforts to work with them simultaneously. In most of the GMDH-type neural networks, neurons in each layer are only connected to a neuron in the adjacent layer, as was the case in Methods I and II previously reported in Nariman-Zadeh et al. (2003). Considering this, it was possible to present a

simple encoding scheme for the genotype of each individual in the population. The encoding schemes in general structure GMDH-type neural networks (GS-GMDH) must, however, demonstrate the ability to represent the different lengths and sizes of such neural networks. In Fig. 2, the neuron *ad* in the first hidden layer is connected to the output layer by directly going through the second hidden layer. Therefore, it is now very easy to notice that the name of output neuron (network's output) includes *ad* twice as *abbcadad*. In other words, a virtual neuron named *adad* has been constructed in the second hidden layer and used with *abbc* in the same layer to make the output neuron *abbcadad*, as shown in Fig. 2.

Such repetition occurs whenever a neuron passes some adjacent hidden layers and connects to another neuron in the next 2nd, or 3rd, or 4th, or ... the following hidden layer. In this encoding scheme, the number of repetitions of that neuron depends on the number of hidden layers, $\tilde{n}$, and is calculated as $2^{\tilde{n}}$. It is easy to realize that a chromosome such as *abab bcbc*, unlike chromosome *abab acbc* for example, is not valid in GS-GMDH networks and has to be simply re-written as *abbc* (Jamali et al., 2009).

The genetic operators of crossover and mutation can now be implemented to produce two offspring from two parents. The natural roulette wheel selection method is used to randomly choose the two parents for the two offspring. The incorporation of a genetic algorithm into the design of such GMDH-type neural networks allows each network to be represented as a string of concatenated sub-strings of alphabetical digits. The fitness, Φ of each entire string of symbolic digits which represents a GMDH-type neural network model is evaluated in the following form (Atashkari et al., 2007).

$$\Phi = 1/E \tag{27}$$

where E is the mean square of error given by Eq. (21), which is minimized through the evolutionary process by maximizing the fitness, Φ. The evolutionary process starts by randomly generating an initial population of symbolic strings, each as a candidate solution. Then, using the genetic operations of roulette wheel selection, crossover, and the mutation of the entire population of symbolic strings improve gradually. In this way, GMDH-type neural network models with progressively increasing fitness, Φ, are produced until no further significant improvement is achievable (Ardalan et al., 2009).

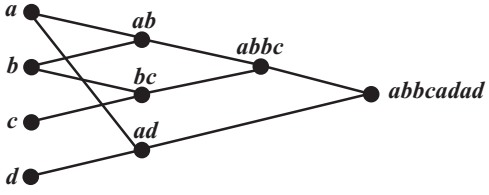

Fig. 2. A GS-GMDH network structure of a chromosome (Nariman-Zadeh et al., 2003).

Table 2
Descriptive statistics of variables.

Variable	Minimum	Mean	Maximum
e_0	0.466	0.794	1.882
ω_n	12.7	29.76	70
LL	25	42.22	79
PI	5	20.65	51
G_s	2.43	2.64	2.8
C_r	0.0088	0.034	0.129

Fig. 3. Descriptive data collection location.

4. Database compilation

The databases include data compiled from 344 consolidation tests for soils sampled at 115 construction sites in province of Mazandaran, Iran (Geotechnical Report of Site Investigation). Following the previous trend of studies, in this study the C_r of the soils was assumed to be affected by the initial void ratio (e_0), natural water content (ω_n), liquid limit (LL), plastic index (PI) and specific gravity (G_s) as summarized in Table 2. The compiled database contains 344 data produced by the Technical and Soil Laboratory of Mazandaran Province of Iran (Fig. 3) which is one of the most experienced consultants in the country, as summarized in Fig. 4. The samples were all collected using a standard procedure and tests were carried out using ASTM D2435 (2011).

5. Evaluation of C_r using GMDH

The main objective of this research is to develop a polynomial function of input geotechnical parameters to estimate C_r as output. In order to demonstrate the prediction ability of the evolved GMDH-type neural network, the data set was divided randomly into two separate data set: the training data set and the testing data set. The aim of the testing data set is to obtain a more generalized model, but it is not incorporated in the training procedure. According to an accepted rule, training and testing data sets must be similar in terms of their statistical properties such as mean and standard deviation (Tokar and Johnson, 1999).

In this study, among 344 data sets, 44 randomly collected data sets were used in the testing stage and 300 data sets were used in the training stage (Table 3).

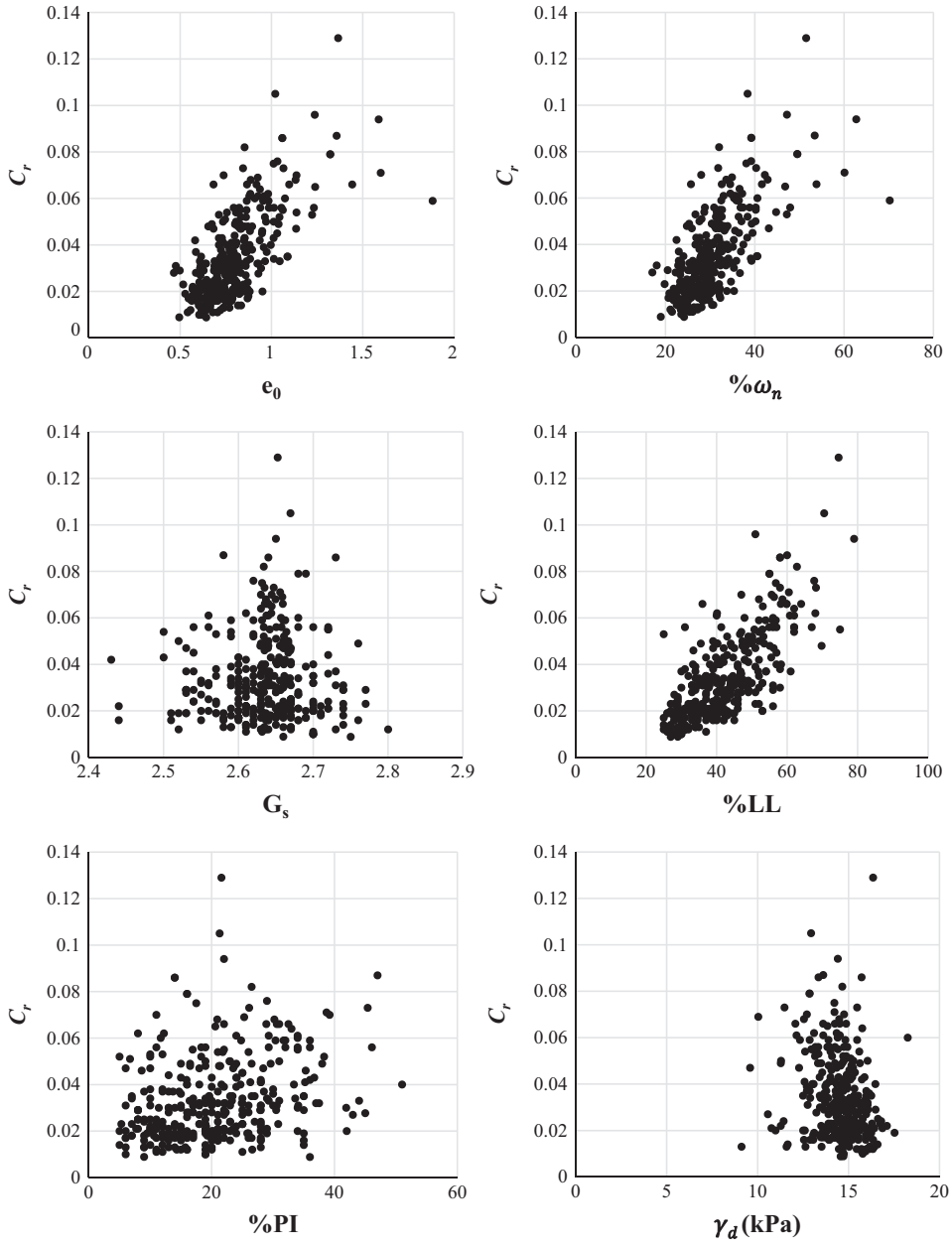

Fig. 4. Descriptive data distribution.

Table 3
Descriptive statistics of variables used in the GMDH.

Variable		Train (300 data set)			Test (44 data set)		
		Minimum	Mean	Maximum	Minimum	Mean	Maximum
Input	e_0	0.466	0.79	1.882	0.517	0.82	1.237
	ω_n	12.7	29.58	70	19.5	30.96	49.2
	LL	25	42	79	26	43.69	68.2
	PI	5	20.5	51	5	21.75	45.2
	G_s	2.43	2.64	2.8	2.52	2.64	2.73
Output	C_r	0.0088	0.033	0.129	0.01	0.039	0.096

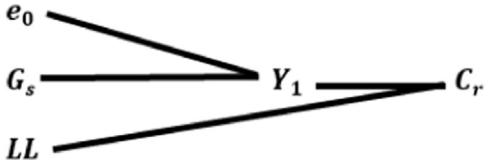

Fig. 5. Evolved structure of generalized GMDH neural network.

In order to overcome the different magnitudes of the input variables, data need to be normalized. Normalization can be done by several equations. In this work, data is scaled between 0.1 and 0.9 as follows (Mehdizadeh and Movagharnejad,

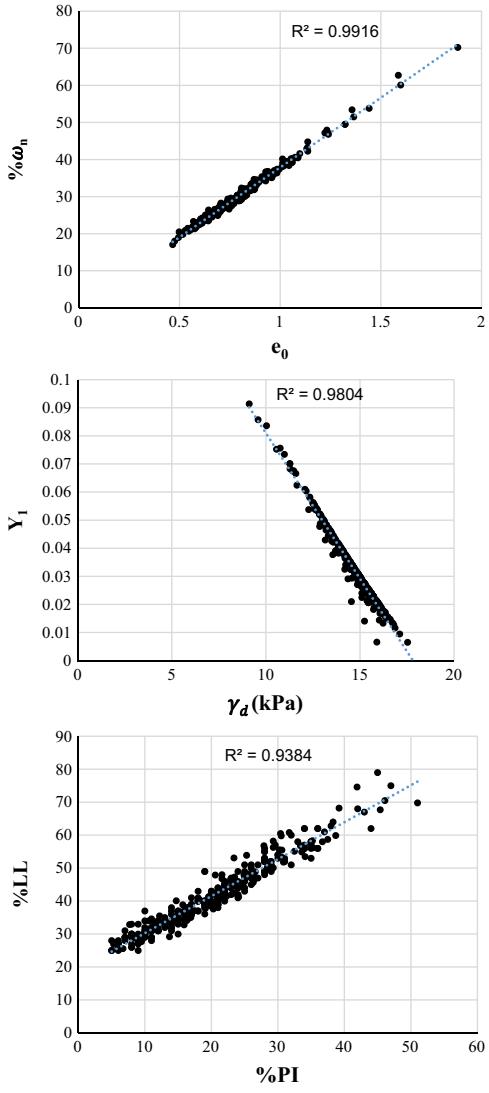

Fig. 6. Evolved structure for γ_d , ω_n and *PI*.

2011):

$$(\text{Scaled})_{\text{value}} = \frac{(\text{Actual})_{\text{value}} - \min_{(\text{Actual value})}}{\max_{(\text{Actual value})} - \min_{(\text{Actual value})}} * 0.8 + 0.1 \quad (28)$$

Various parameters involved in the GMDH predictive such as population size, number of generations, number of hidden layers, crossover probability and mutation probability. The selection of parameters will affect the model generalization capability of GMDH. To genetically design such a GMDH-type neural network, a population of 100 individuals with a crossover probability of 0.95 and mutation probability of 0.01 was used in 300 generations for the population size with no further improvements made. A number of hidden layers can be tried to achieve the best relative fit. In this study single, double and triple layers were examined, and although the three-layer provided a slightly better fit, it was decided to opt for a single layer to avoid over-fitting and to obtain a simpler correlation. It is noteworthy that the inherent inaccuracies of the test data and the uncertainties associated with experimentation prohibit an exact fit, and only the general trend is determinable. The structure of the evolved GMDH-type neural network is shown in Fig. 5 corresponding to the genome representations: Initial void ratio (e_0), liquid limit (*LL*) and specific gravity (G_s), respectively.

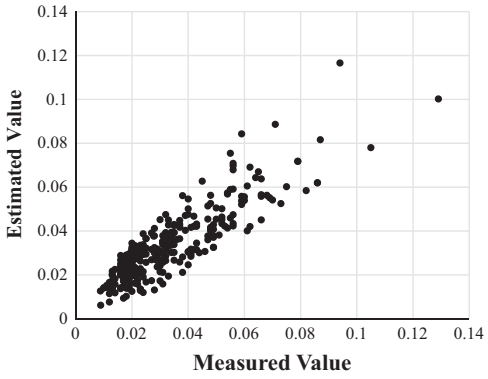

Fig. 7. The measured C_r obtained from the consolidation test versus the GMDH estimated C_r (result of training process).

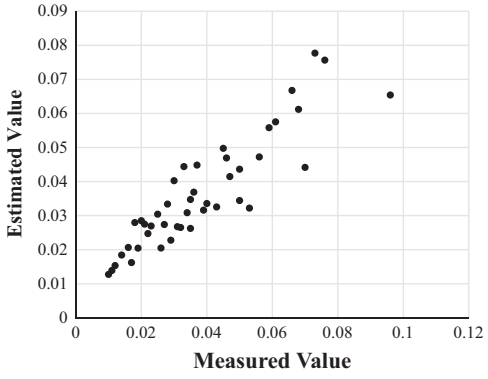

Fig. 8. The measured C_r obtained from the consolidation test versus the GMDH estimated C_r (result of testing process).

Of note, dry density (γ_d), natural water content (ω_n) and plastic index (*PI*) are linearly related to the hidden layer parameter (Y_1), e_0 and *LL* respectively (Fig. 6). Thus these three variables do not appear in the GMDH structure.

The corresponding polynomial representation of such model for C_r is as follows:

$$C_r = -0.0115 + 0.587\ Y_1 + 0.00017\ LL + 0.524\ {Y_1}^2 + 0.000008\ LL^2 + 0.0015\ LL.\ Y_1 \quad (29-a)$$

$$Y_1 = -1.689 + 0.125\ e_0 + 1.286\ G_s - 0.027 e_0^2 - 0.251\ G_s^2 + 0.00004\ e_0\ .G_s \quad (29-b)$$

Figs. 7 and 8 show the relationship between output targets and predicted values obtained through the training and testing process. The behavior of the GMDH-type neural network model is also depicted in Fig. 9 for all 344 data set of C_r.

The model shows one of the best correlations for both the training and testing data compared with the conventional empirical formulas and the proposed equation results in points more closely located around the 1:1 line. It can be seen that the evolved GMDH-type neural network could successfully model

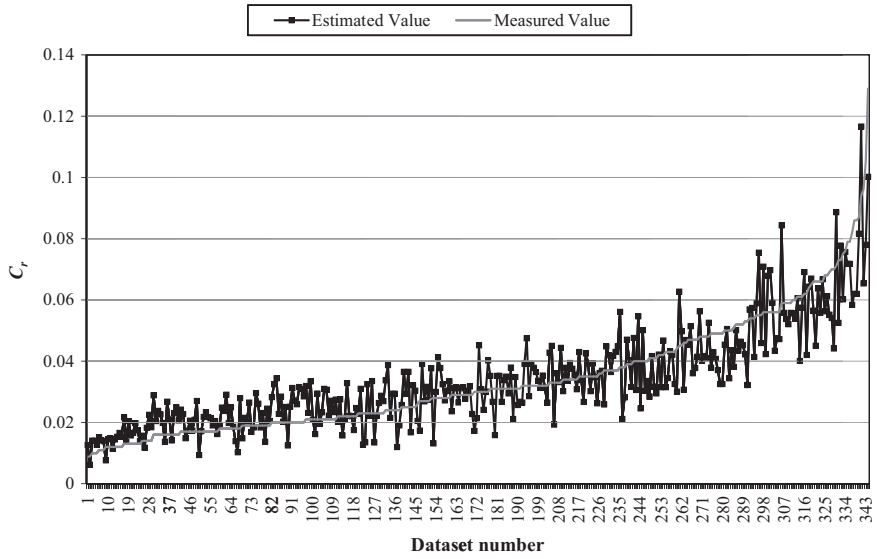

Fig. 9. Comparison of measured values of C_r with the estimated values using evolved GMDH neural network for all 344 data set.

and predict the output of testing data that was not used during the training process.

The absolute fraction of variance (R^2), root mean squared error (RMSE), mean absolute percent error (MAPE) and mean absolute deviation (MAD) were used to evaluate the performance of the proposed equations and models, which are defined as follows:

$$R^2 = 1 - \left[\frac{\sum_1^M (C_{mi} - C_{pi})^2}{\sum_1^M (C_{mi})^2}\right] \tag{30}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{M}\sum_1^M (C_{mi} - C_{pi})^2} \tag{31}$$

$$\mathrm{MAPE} = \frac{\sum_1^M |C_{mi} - C_{pi}|}{\sum_1^M C_{mi}} \times 100 \tag{32}$$

$$\mathrm{MAD} = \frac{\sum_1^M |C_{mi} - C_{pi}|}{M} \tag{33}$$

where C_{pi} and C_{mi} are the predicted and the measured C_r respectively.

The lower the RMSE, MAPE and MAD values, the better the model performance. Under ideal conditions, an accurate and precise method gives R^2 of 1.0, RMSE, MAPE and MAD of 0.

In Table 4, the predictability of the GMDH model is statistically compared with the empirical formulas. The value of RMSE, MAPE and MAD are found for the GMDH model in both training and testing stage.

6. Comparison of existing equations

In order to prepare a platform for comparison, the collected data sets were used to examine the accuracy of the empirical correlations proposed by other researchers (Table 1) and the RMSE, MAPE, MAD and the R^2 values were determined and compared against GMDH model. Furthermore, the coefficients of the empirical correlations were modified to maximize the R^2 value. A summary of the results is presented in Table 5.

Table 4
Statistical results for GMDH.

Cr, Model	R^2	MAPE	RMSE	MAD
C_r , GMDH (training)	0.954	19.15	0.008	0.0069
C_r , GMDH (testing)	0.956	18.27	0.008	0.0066

It can be seen that the best fit amongst the previously proposed correlations are achieved by Nakase et al. (1988) and Sinan (2009) (Eqs. 9 and 10). By modifying the coefficients of existing relationships a slightly better fit is achieved (Eqs. 34–41). Note, however, the least disparity in C_r prediction is achieved by the GMDH approach.

A graphic representation of the comparative accuracy of the various correlations and the proposed method is shown by plotting the Scaled Relative Error (E_r) versus Scaled Cumulative Frequency (SCF), where

$$E_r = 100\left(C_{pi} - C_{mi}\right)/C_{mi} \tag{42}$$

As is seen in Fig. 10, a broader range of predictions is given by almost all of the empirical relationships in comparison to GMDH approach. The proposed GMDH model is distinctly more accurate in comparison with most of the other empirical correlations and the modified correlations (Table 5).

7. Sensitivity analysis

The sensitivity analysis of the obtained model is performed to evaluate the input parameters influence on the model output.

The sensitivity test is carried out by varying each of the normalized input neurons at a constant rate, one at a time, while the other variables are kept constant. Various constant rates (0.75, 0.80, 0.85,… , 1.25) are selected in the study. For

Table 5
Statistical results for conventional empirical formulas (C_r).

Equation	Eq. no.	R^2	MAPE	RMSE	MAD
$C_r = 0.0003(\omega_n + 7)$	(4)	0.45	67.44	0.028	0.228
$C_r = 0.015(e_0 + 0.007)$	(5)	0.49	64.5	0.027	0.021
$C_r = 0.041 - 0.0268\ e_0$	(6)	0.54	51.89	0.0255	0.0172
$C_r = 0.045 - 0.0283\ e_0$	(7)	0.58	50.08	0.0243	0.0166
$C_r = 0.126 - 0.115\ e_0$	(8)	0.39	81.44	0.0359	0.0269
$C_r = 0.00194(PI - 4.6)$	(9)	0.85	33.5	0.0145	0.0111
$C_r = 0.0007\ LL + 0.0062$	(10)	0.86	34.02	0.0139	0.0113
$C_r = 0.1257\ \gamma_d^{-2.8826}$	(11)	0.82	39.8	0.0164	0.0135
$C_r = 0.126(e_0 + 0.003\ LL - 0.06)$	(12)	0.45	67.98	0.028	0.023
$C_r = 0.0133\ e^{0.036\ \omega_n}$	(13)	0.46	67.5	0.0283	0.0229
$C_r = 0.000463\ LL\ G_s$	(14)	0.69	57.2	0.0216	0.0194
$C_r = -0.000319\ \omega_n - 0.027277\ \gamma_d + 0.064019\ e_0 + 0.037$	(15)	0.52	70.74	0.0268	0.024
This study correlation (GMDH)	(29)	0.95	18.83	0.008	0.0068
This study correlation ($C_r = -0.025 + 0.002\omega_n$)	(34)	0.86	32.17	0.0142	0.0109
This study correlation ($C_r = -0.024 + 0.0732\ e_0$)	(35)	0.88	29.38	0.0132	0.0099
This study correlation($C_r = 0.0048 + 0.001\ PI$)	(36)	0.87	30.5	0.0135	0.0103
This study correlation ($C_r = -0.0214 + 0.0013\ LL$)	(37)	0.89	28.27	0.0125	0.009
This study correlation($C_r = 65.06\ \gamma_d^{-2.84}$)	(38)	0.87	31.05	0.0137	0.0105
This study correlation ($C_r = 0.000213\omega_n^{1.486}$)	(39)	0.86	31.54	0.014	0.0106
This study correlation ($C_r = -0.049 + 0.052\ e_0 + 0.001\ LL$)	(40)	0.89	26.68	0.0124	0.009
This study correlation ($C_r = 0.095 + 0.03e_0 - 0.000025LL - 0.0059\gamma_d$)	(41)	0.88	29.2	0.013	0.009

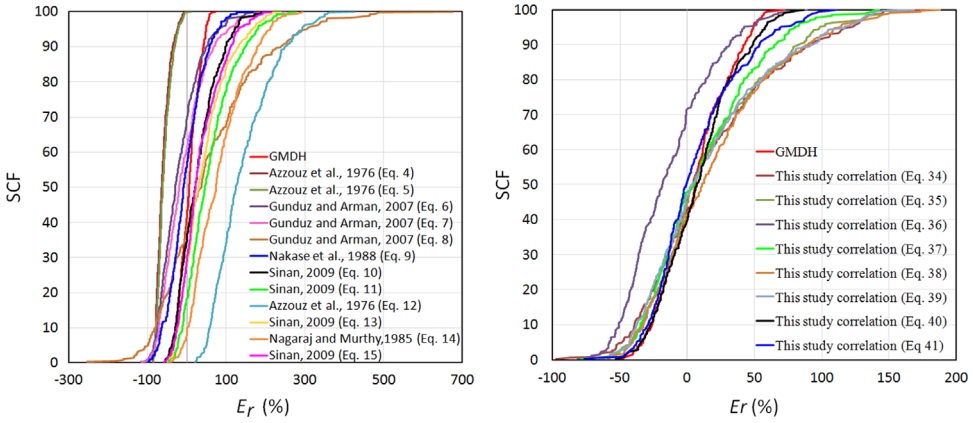

Fig. 10. Scaled relative errors of C_r estimated.

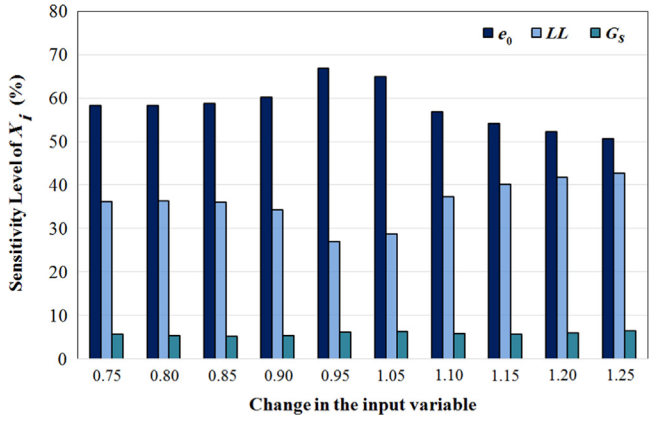

Fig. 11. Results of the sensitivity analysis of the obtained model for C_r.

every input neuron, the percentage change in the output, as a result of the change in the input neuron, is observed. The sensitivity of each input neuron is calculated by the following (Liong et al., 2000):

$$\text{Sensitivity level of } X_i(\%) = \frac{1}{M}\sum_{j=1}^{M}\left(\frac{\%\text{ change in output}}{\%\text{ change in input}}\right)_j \times 100 \quad (43)$$

Results of the sensitivity analysis of the obtained model are shown in Fig. 11. It can be noticed from Fig. 11 that C_r is considerably influenced by e_0 and its value decreases by increasing e_0 value. Also, G_s has a little effect on the

correlation.

8. Conclusions

Empirical correlations provide a means for the quick evaluation of engineering parameters and the verification of test results. However, ordinary regression analyses may miss the inter-dependency of variables, and tend to be inefficient in cases of multi-parametric problems such as the one at hand.

In this study, the performances of widely used single and multi-variable empirical equations for the estimation of the recompression index were evaluated using a database consisting of 344 wide-ranging samples from the province of Mazandaran, Iran. Using the same database, an attempt was made to predict this index using a neural network simulation. A sensitivity analysis on the obtained model was also used to study the influence of various parameters.

The results indicate the following:

1. Amongst the previously proposed equations, the equations proposed by Nakase et al. (1988) and Sinan (2009) gave the lowest RMSE, MAPE, MAD and the highest R^2 value.
2. The liquid limit appears to have the most influence on the C_r value, since amongst the single parameter correlations, the best fit is achieved by the one that utilizes the liquid limit.
3. The developed GMDH-type neural network model was proven to be more efficient than all of the empirical correlations derived from an ordinary regression analysis.

It may thus be concluded that since all experimental data are prone to inaccuracies and uncertainties, a global pattern recognition technique such as GMDH with no partial perspective achieves a more accurate prediction than an ordinary regression analysis.

Through this study it was determined that the most influential physical characteristics of soils on C_r are the liquid limit and the initial void ratio.

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